ME 321: Fluid Mechanics-I

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Lecture - 09 (28/06/2025)
Fluid Dynamics: Applications of Bernoulli Equation

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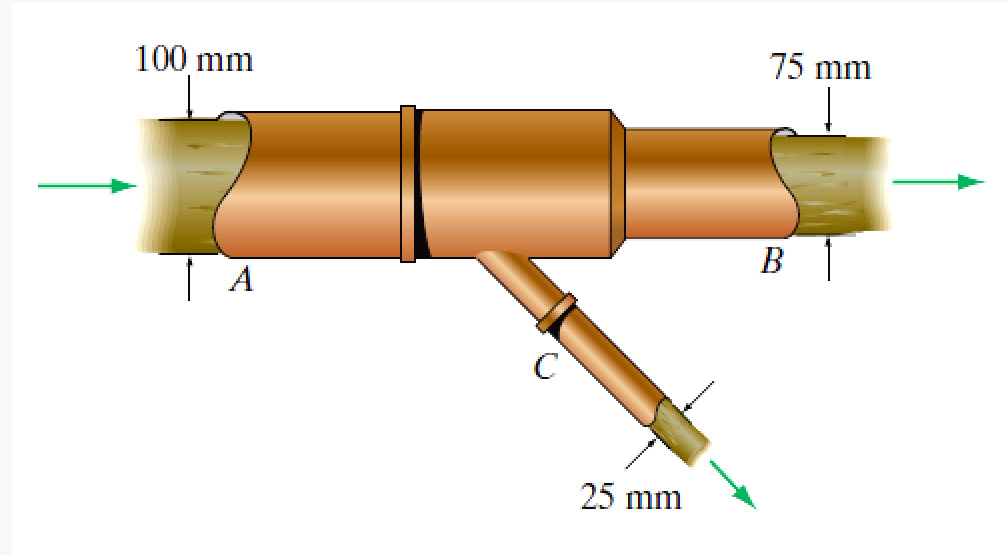
Problem # 1

Oil (SG = 0.88) flows through the horizontal pipe under a pressure of 400 kPa and at a velocity of 2.5 m/s at A. Determine the pressure in the pipe B if the pressure at C is 150 kPa. Neglect any elevation difference.



Solution:

From continuity equation, (steady flow)

$$\cancel{\frac{d}{dt}}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right)dA = 0$$

$$\Rightarrow \int_{CS}\rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right)dA = 0$$

$$\Rightarrow -\rho A_A V_A + \rho A_B V_B + \rho A_C V_C = 0$$

Not necessarily you have to start from the first relation, but you have to realize that **Eq. (A)** is the result of the integral form of continuity equation for steady inviscid incompressible flows.

$$\boxed{\Rightarrow A_A V_A = A_B V_B + A_C V_C} \qquad \textbf{(A)}$$

$$\Rightarrow \frac{\pi}{4}d_A^2 V_A = \frac{\pi}{4}d_B^2 V_B + \frac{\pi}{4}d_C^2 V_C$$

$$\Rightarrow \frac{\pi}{4}(0.1)^2(2.5) = \frac{\pi}{4}(0.075)^2 V_B + \frac{\pi}{4}(0.025)^2 V_C$$

$$\boxed{\Rightarrow 9V_B + V_C = 40 \qquad (i)}$$

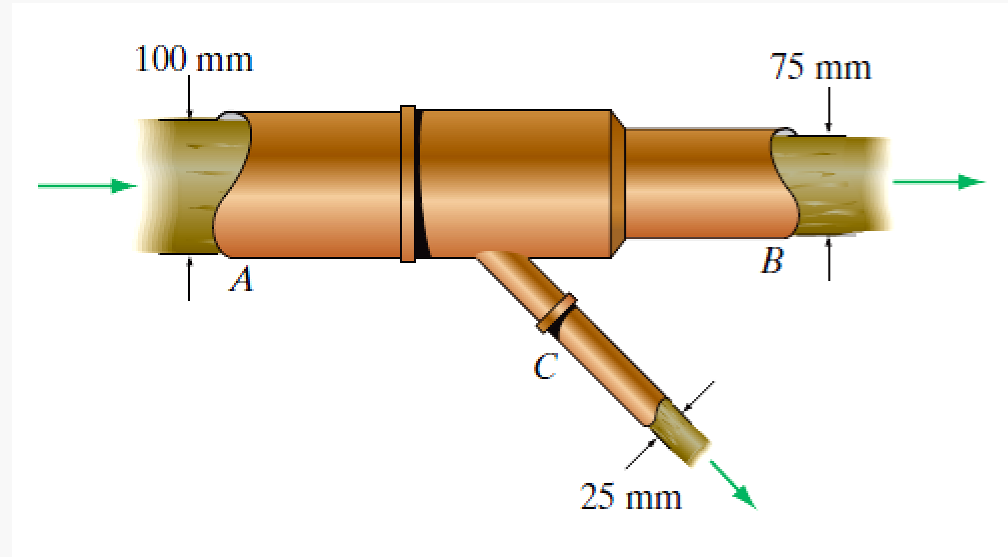
Problem # 1

Apply Bernoulli equation between point A and C

$$\frac{p_A}{\gamma}+\frac{V_A^{\,2}}{2g}+z_A=\frac{p_C}{\gamma}+\frac{V_C^{\,2}}{2g}+z_C$$

$$\Rightarrow \frac{400\times10^3}{(880\times9.81)}+\frac{(2.5)^2}{2g}+\cancel{z_A}=\frac{150\times10^3}{(880\times9.81)}+\frac{V_C^2}{2g}+\cancel{z_C}$$

$$\boxed{\Rightarrow V_C=23.97\text{ m/s}}$$



Now use Eq. (i)

$$9V_B+V_C=40 \qquad \rightarrow \quad V_B=1.78\text{ m/s}$$

Again, apply Bernoulli equation between point A and B

$$\frac{p_A}{\gamma}+\frac{V_A^{\,2}}{2g}+z_A=\frac{p_B}{\gamma}+\frac{V_B^{\,2}}{2g}+z_B$$

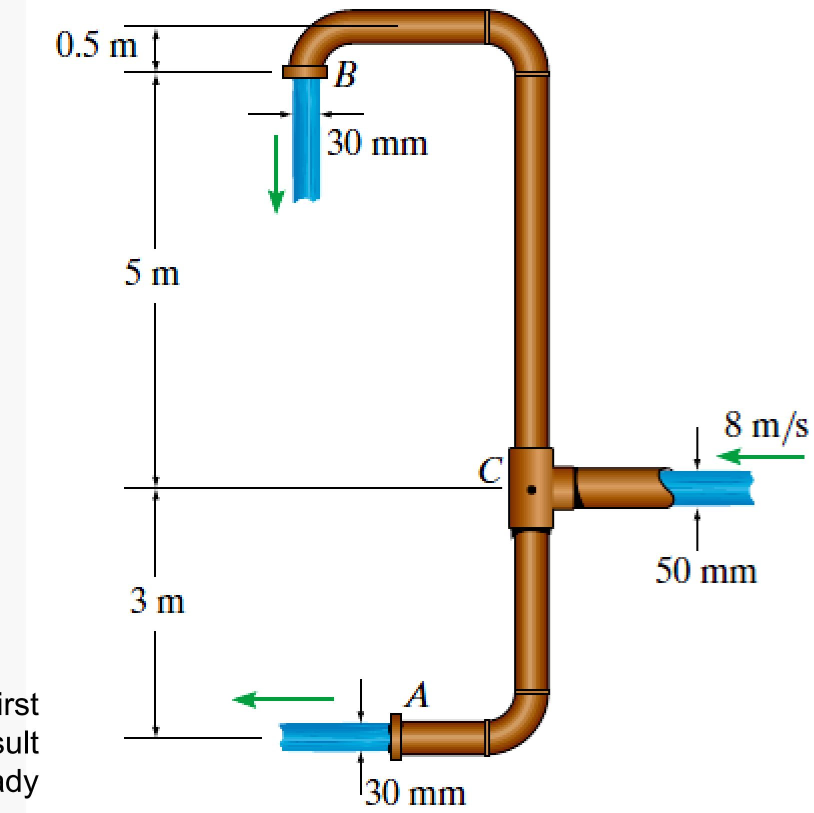
$$\Rightarrow \frac{400\times10^3}{(880\times9.81)}+\frac{(2.5)^2}{2g}+\cancel{z_A}=\frac{p_B}{(880\times9.81)}+\frac{(1.78)^2}{2g}+\cancel{z_C}$$

$$\boxed{\Rightarrow p_B=401.4\text{ kPa}}$$ **Ans.**

ratio of pressure to velocity heads??

Problem # 2

Determine the velocity of the flow out of the vertical pipes at A and B, if water flows into Tee at 8 m/s and under a pressure of 40 kPa.



Solution:

From continuity equation, (steady flow)

$$\cancel{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall} + \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right) dA = 0$$

$$\Rightarrow \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right) dA = 0$$

$$\Rightarrow -\rho A_C V_C + \rho A_A V_A + \rho A_B V_B = 0$$

Not necessarily you have to start from the first relation but you have to realize that **Eq** is the result of the integral form of continuity equation for steady incompressible flows.

$$\Rightarrow A_C V_C = A_A V_A + A_B V_B$$

$$\Rightarrow \frac{\pi}{4} d_C^2 V_C = \frac{\pi}{4} d_A^2 V_A + \frac{\pi}{4} d_B^2 V_B$$

$$\Rightarrow \frac{\pi}{4}(0.05)^2(8) = \frac{\pi}{4}(0.03)^2 V_A + \frac{\pi}{4}(0.03)^2 V_B$$

$$\boxed{\Rightarrow V_A + V_B = 22.22 \qquad (i)}$$

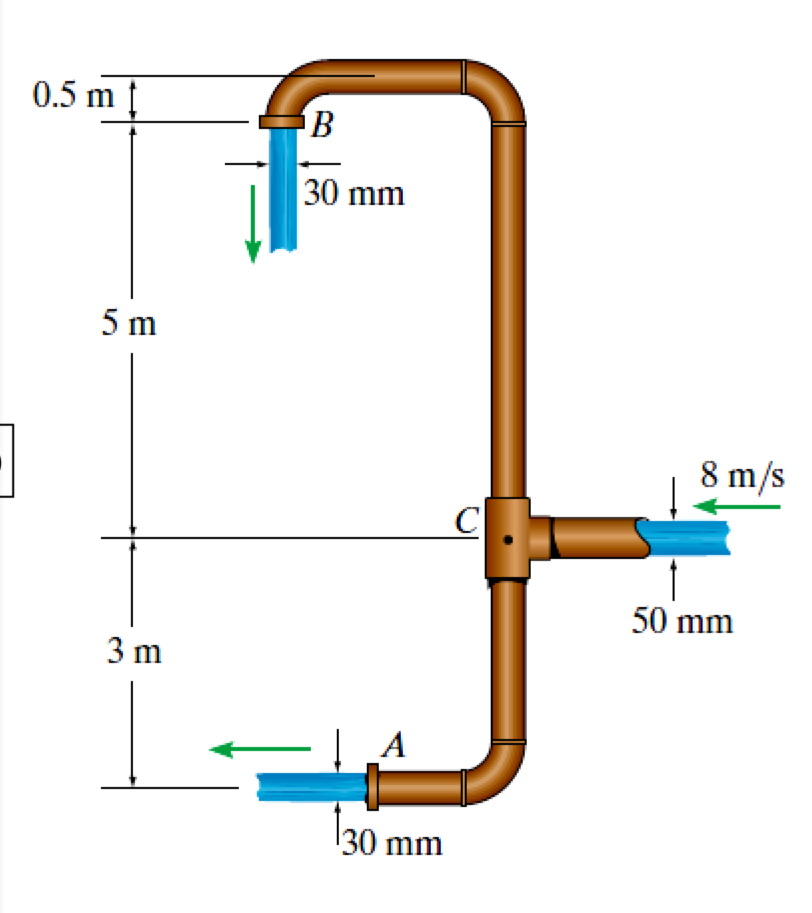
Problem # 2

Apply Bernoulli equation between point C and A

$$\frac{p_C}{\gamma} + \frac{V_C^{\,2}}{2g} + z_C = \frac{p_A}{\gamma} + \frac{V_A^{\,2}}{2g} + z_A$$

$$\Rightarrow \frac{40\times10^3}{(1000\times9.81)} + \frac{(8)^2}{2g} + 0 = \frac{0}{(1000\times9.81)} + \frac{V_A^2}{2g} - 3 \qquad \boxed{;\ p_A = p_B = 0\ \ \text{(open discharge)}}$$

$$\boxed{\Rightarrow V_A = 14.24 \text{ m/s}}$$ **Ans.**

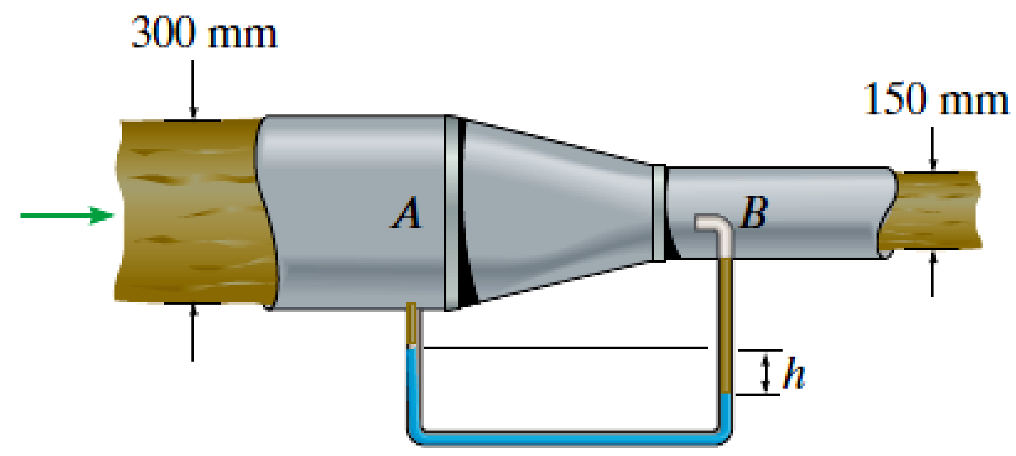


Now use Eq. (i)

$$\boxed{V_A + V_B = 22.22 \qquad \rightarrow \qquad V_B = 7.98 \text{ m/s}}$$ **Ans.**

Problem # 3

Determine the difference in height h of the water column in the manometer if the flow of oil through the pipe is 0.04 m^3/s. Take ρ_{oil} = 875 kg/m^3



Solution:

From continuity equation, (steady flow)

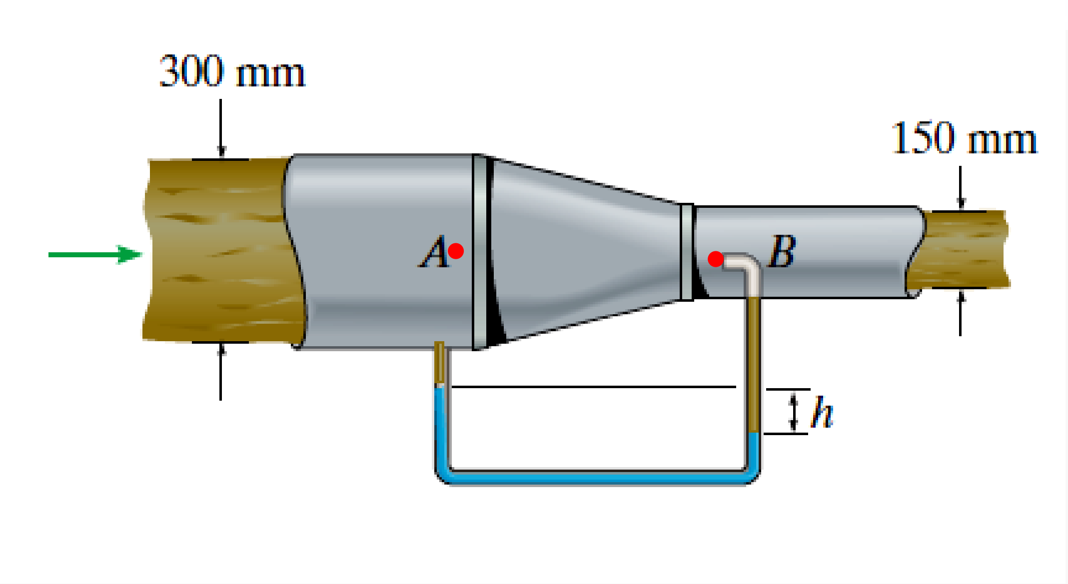
$$Q = A_A V_A = 0.04 \quad \text{m}^3/\text{s} \qquad \text{(given)}$$

$$\therefore \quad V_A = \frac{0.04}{\frac{\pi}{4} d_A^{\,2}} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.57 \quad \text{m/s}$$

$$B \text{ is the stagnation point} \quad \therefore \; V_B = 0$$

Problem # 3

Apply Bernoulli equation between point A and B



$$\frac{p_A}{\gamma_{oil}}+\frac{V_A^{\,2}}{2g}+z_A=\frac{p_B}{\gamma_{oil}}+\frac{V_B^{\,2}}{2g}+z_B$$

$$\Rightarrow \frac{p_A}{\gamma_{oil}}+\frac{(0.57)^2}{2g}+\cancel{z_A}=\frac{p_B}{\gamma_{oil}}+\frac{0^2}{2g}+\cancel{z_B}$$

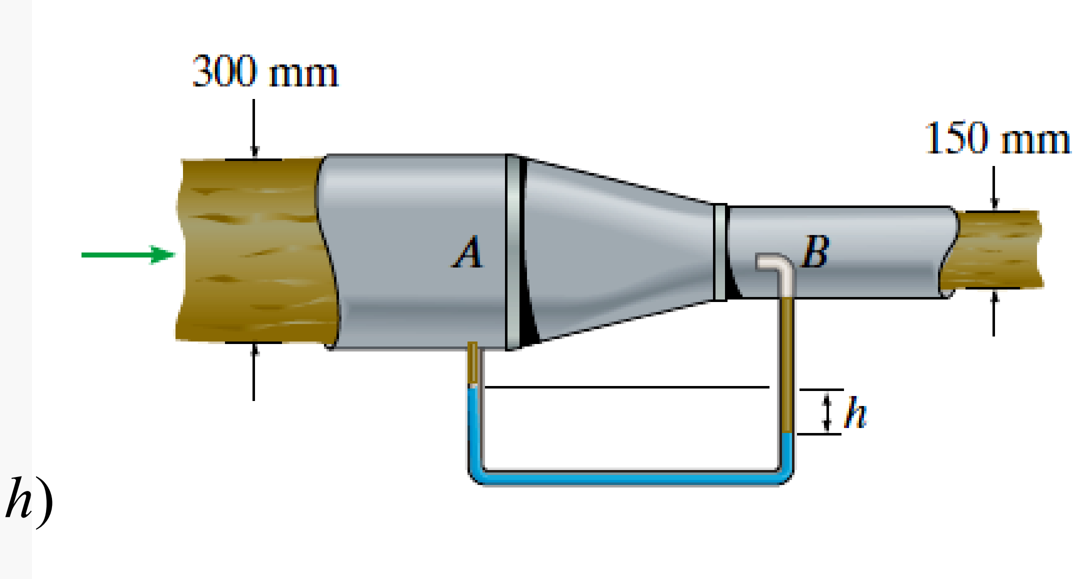
$$\Rightarrow p_B-p_A=\frac{(0.57)^2}{2g}\gamma_{oil}$$

$$\Rightarrow p_B-p_A=\frac{(0.57)^2}{2g}(\rho_{oil}g)$$

$$\Rightarrow p_B-p_A=\frac{(0.57)^2}{2}(875)$$

$$\boxed{\Rightarrow p_B-p_A=142.1 \qquad (i)}$$

Problem # 3



From **principle of manometry (fluid statics)**

$$p_A + \rho_{oil} g h_{AC} + \rho_{water} g h_{CD} = p_B + \rho_{oil} g h_{BD}$$

$$\Rightarrow p_A + (875)(9.81)a + (1000)(9.81)h = p_B + (875)(9.81)(a+h)$$

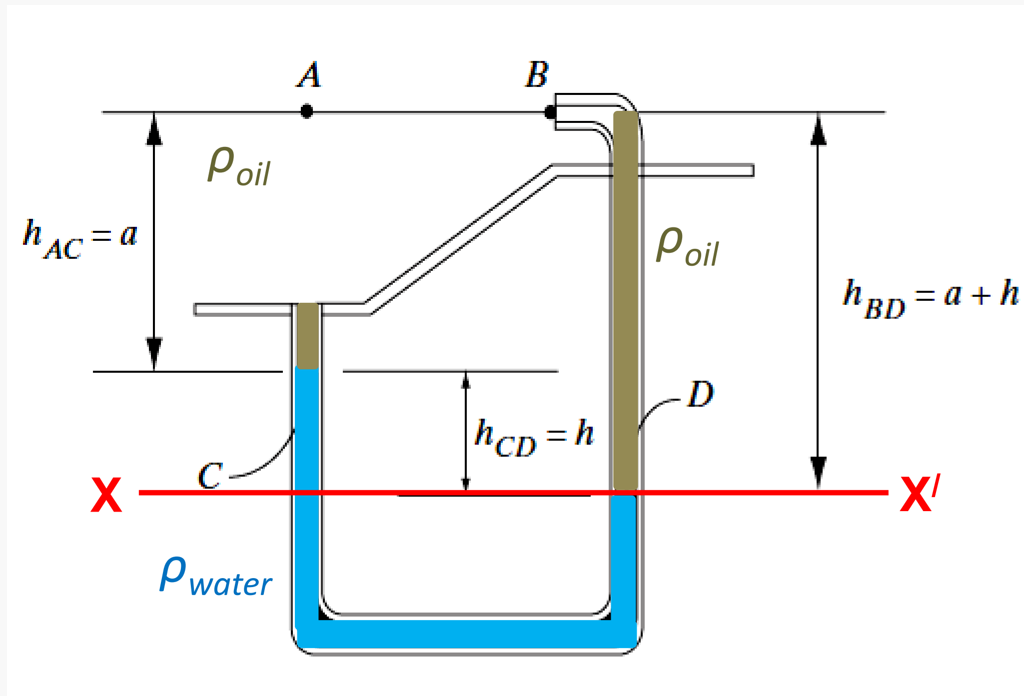
$$\Rightarrow p_B - p_A = 1226.25\,h$$

$$\Rightarrow 142.1 = 1226.25\,h$$

From Eq. (i): $\boxed{\Rightarrow p_B - p_A = 142.1}$

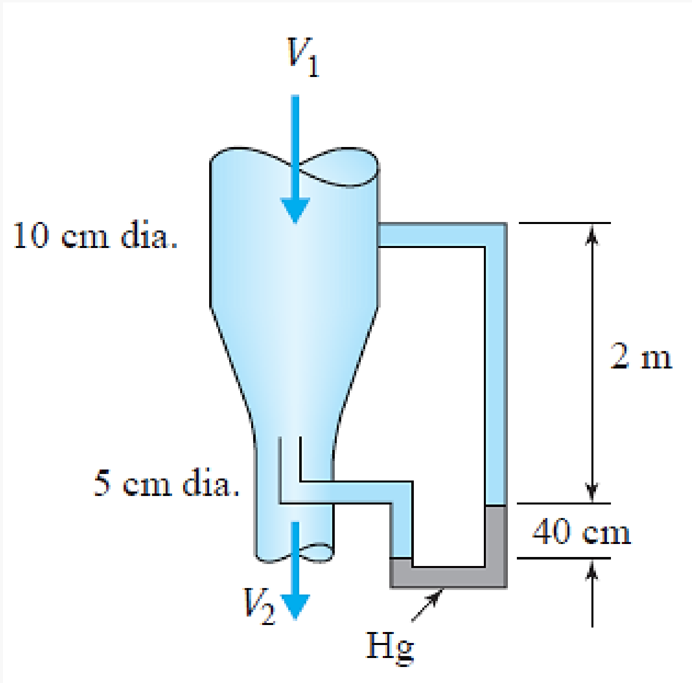
$$\boxed{\begin{aligned} &\Rightarrow h = 0.116\,\text{m} \\ &\therefore h = 116\,\text{mm} \end{aligned}}$$

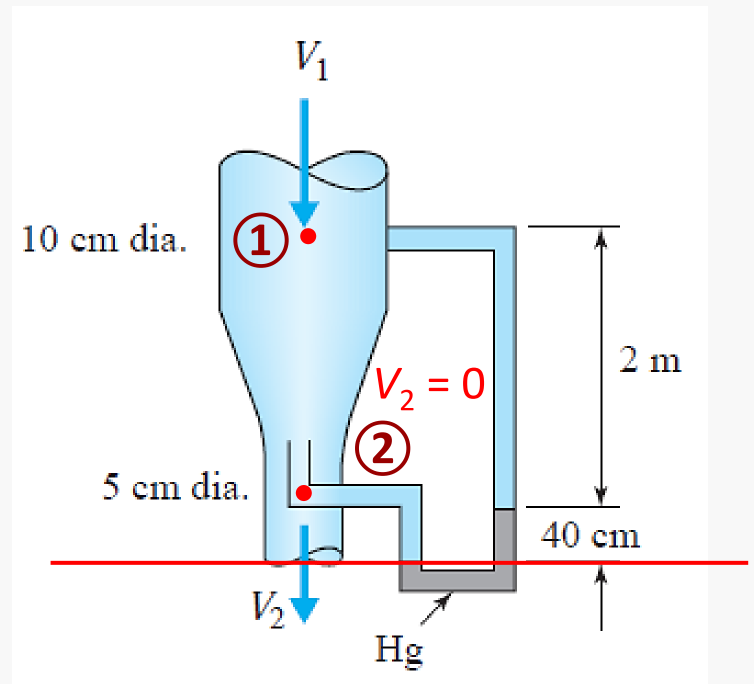
Ans.



Problem # 4

Find the velocity V_1 of the water in the vertical pipe shown in Fig. Assume no losses.



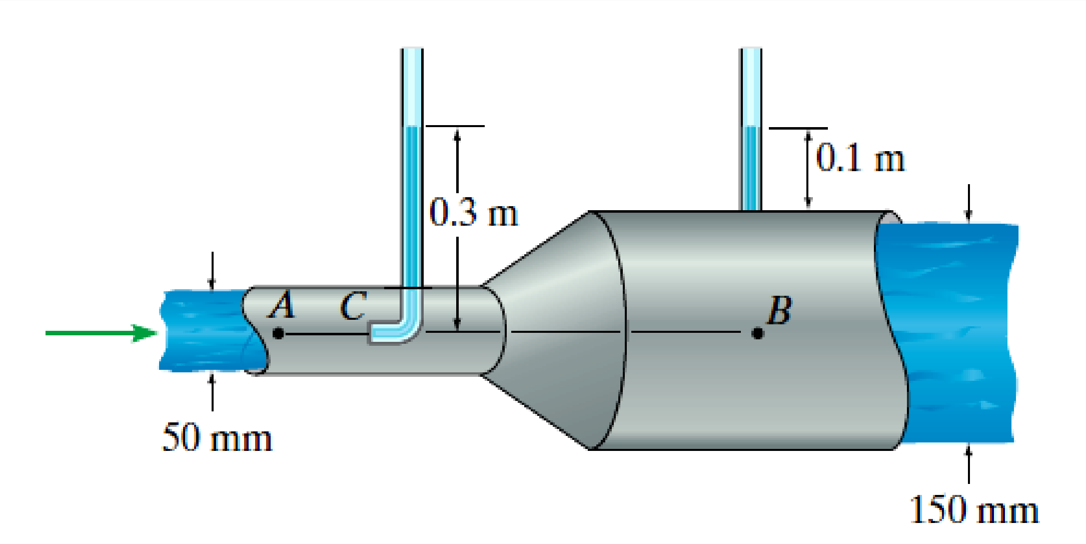


(1) Apply Bernoulli Equation between 1 and 2
(2) Use the principle of manometry

Ans: $V_1 = 9.94$ m/s

Problem # 5

Determine the volumetric flow rate of water and the pressure in the pipe at A if the height of the water column in the Pitot tube is 0.3 m and the height in the piezometer is 0.1 m.



Solution:

Use continuity equation between points A and B $\boxed{V_A = 9V_B}$

Calculate pressure at B $\boxed{p_B = 1.71675(10^3)\,\text{Pa}}$

Calculate stagnation pressure at C $\boxed{p_C = 2.943(10^3)\,\text{Pa}}$

Use Bernoulli equation between points **A** and **C** $\boxed{p_A + 500V_A^2 = 2.943(10^3)}$

Use Bernoulli equation between points **C** and **B** $\boxed{V_B = 1.566\,\text{m/s}}$

$$\boxed{Q = A_A V_A = A_B V_B}$$

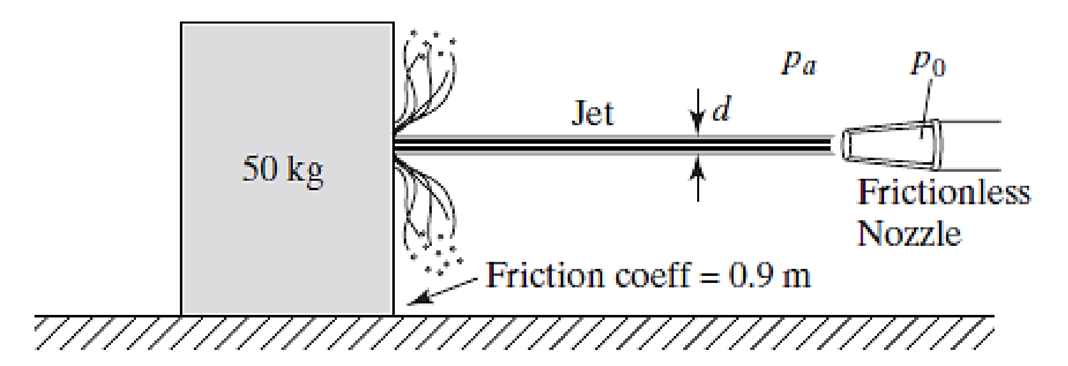
Ans.

$$p_A = -96.4\,\text{kPa}$$

$$Q = 0.0277\,\text{m}^3/\text{s}$$

Problem # 6

A water jet hits horizontally a 50-kg block. The friction coefficient between the block and the ground is 0.9. What is the minimum diameter d of the water jet for the block to slide to the left?
Assume the stagnation pressure inside the nozzle is p_0 = 4 atm the pressure outside the nozzle is p_a = 1 atm (1 atm= 1.013×10^5 N/m^2), and the flow is ideal through the nozzle.



Problem # 7

The neighbor kid decided to water the garden while standing on his skateboard.
As he opened the valve, the circular jet speed was 7 m/s and its diameter was d_2 = 20 mm. If his total mass is 40 kg, then calculate the force of the jet and his initial acceleration (assuming no friction).